VERTEX DEGREE OF CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY GRAPH

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ABSTRACT – A new intuitionistic fuzzy graph can be generated by the use of the cartesian product of two intuitionistic fuzzy graphs. Degree of vertices in an intuitionistic fuzzy graph gives a complicated picture when we consider the cartesian product for intuitionistic fuzzy graphs with large number of vertices. We propose a methodology to find the degree of vertices for the cartesian product of intuitionistic fuzzy graph from the degree of vertices of two intuitionistic fuzzy graphs under certain conditions. This methodology simplifies the process and the outcome of the newly generated intuitionistic fuzzy graphs. These concepts are analyzed through suitable illustrations.

KEYWORDS - Intuitionistic fuzzy graph, degree of vertex in IFG, cartesian product.

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1 INTRODUCTION

A mathematical frame work to describe the phenomena of uncertainty in real life situation has been suggested by Zadeh in 1965[9]. The theory of fuzzy graphs was independently developed by Rosenfeld[11], yeh and bang [10] in 1975. In 1983, Atanassov[1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets[9]. Fuzzy set give the degree of membership of an element in the given set, while intuitionistic fuzzy set gives both the degree of membership and non membership which are more or less independent from each other. The only condition is that the sum of these two degrees should not exceed 1. In [3] Karunambigai M. G. and Parvathi R. introduces intuitionistic fuzzy graph as a special case of Atanassov's IFG. The operations on IFG was introduces by R. Parvathi, M. G. Karunambigai and K. Atanassov [4]. Degree, Order and Size in IFG was introduced by A. Naggor Gani and S. Shajitha Begum[5]. The degree of a vertex in some fuzzy graphs was introduced by A. Nagooor Gani and K. Radha[2]. In this paper, we find the degree of vertices for the cartisian product of intuitionistic fuzzy graph from the degree of vertices of two intuitionistic fuzzy graph under certain conditions. First we go through some of the basic definitions in intuitionistic fuzzy graphs.

2 PRELIMENARIES

2.1 Definition

An IFG is of the form G: (V, E) where

(i) V = {$v_1, v_2, v_3, \dots , v_n$} such that $\mu_1 : V \rightarrow [0,1]$ and $\gamma_1 : V \rightarrow [0,1]$ denote the degree of membership and non - membership of the element $v_i \in V$ respectively,

and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ (1)

for every $v_i \in V, (i = 1, 2, \dots n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \rightarrow [0,1]$ and $\gamma_2 : V \rightarrow [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j) \quad (2)$$

$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j) \quad (3)$$

And $0 \leq \mu_2(v_i) + \gamma_2(v_i) \leq 1$ (4)

For every $(v_i, v_j) \in E, (i, j = 1,2,\dots,n)$.

Notations

The triplets $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and non – membership of the vertex v_i. The triplets $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and non – membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

2.2 Definition

Let G = (V,E) be an IFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v),\ d_\gamma(v))$

where $d_\mu(v) = \sum_{u \neq v} \mu_2(u,v)$ and $d_\gamma(v) = \sum_{u \neq v} \gamma_2(u,v)$.

2.3 Definition

The Cartesian product of two IFGs G_1 and G_2 is defined as a IFG $G = G_1 \times G_2 : (V, E")$ where $V = V_1 \times V_2$ and

$$E'' = \{(u_1,u_2)(v_1,v_2) / u_1 = v_1 \,\&\, u_2v_2 \in E_2 \text{ or } u_2 = v_2 \,\&\, u_1v_1 \in E_1\}$$

with

$$\langle(\mu_1 \times \mu'_1), (\gamma_1 \times \gamma'_1)\rangle(u_1,u_2) = \langle \min(\mu_1(u_1), \mu'_1(u_2)), \max(\gamma_1(u_1), \gamma'_1(u_2))\rangle$$

for every $(u_1,u_2) \in V$

and

$$\langle(\mu_2 \times \mu'_2), (\gamma_2 \times \gamma'_2)\rangle(u_1,u_2)(v_1,v_2) = \begin{cases} \langle \min(\mu_1(u_1), \mu'_2(u_2,v_2)), \max(\gamma_1(u_1), \gamma'_2(u_2,v_2))\rangle & \text{if } u_1 = v_1 \,\&\, (u_2,v_2) \in E_2 \\ \langle \min(\mu'_1(u_2), \mu_2(u_1,v_1)), \max(\gamma'_1(u_2), \gamma_2(u_1,v_1))\rangle & \text{if } u_2 = v_2 \,\&\, (u_1,v_1) \in E_1 \\ \langle 0,0\rangle & \text{otherwise.} \end{cases}$$

3. Degree of vertices in Cartesian product of IFG

$$d_{G_1 \times G_2}(u_1,u_2) = \left\langle d_{\mu_2 \times \mu'_2}(u_1,u_2),\ d_{\gamma_2 \times \gamma'_2}(u_1,u_2)\right\rangle$$

$$= \left\langle \sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_2 \times \mu'_2)(u_1,u_2)(v_1,v_2), \sum_{(u_1,u_2)(v_1,v_2) \in E} (\gamma_2 \times \gamma'_2)(u_1,u_2)(v_1,v_2)\right\rangle$$

$$= \left\langle \sum_{u_1 = v_1,\,(u_2,v_2) \in E_2} \mu_1(u_1) \wedge \mu'_2(u_2,v_2), \sum_{u_1 = v_1,\,(u_2,v_2) \in E_2} \gamma_1(u_1) \vee \gamma'_2(u_2,v_2)\right\rangle + \left\langle \sum_{u_2 = v_2,\,(u_1,v_1) \in E_1} \mu'_1(u_2) \wedge \mu_2(u_1,v_1), \sum_{u_2 = v_2,\,(u_1,v_1) \in E_1} \gamma'_1(u_2) \wedge \gamma_2(u_1,v_1)\right\rangle. \quad \text{-------(1)}$$

3.1 Theorem

Let $G_1 : \langle(v_i, \mu_1, \gamma_1), (e_{ij}, \mu_2, \gamma_2)\rangle$ and $G_2 : \langle(v_i, \mu'_1, \gamma'_1), (e_{ij}, \mu'_2, \gamma'_2)\rangle$ be two IFGs. If $\mu_1 \geq \mu'_2$, $\gamma_1 \leq \gamma'_2$ and $\mu'_1 \geq \mu_2$, $\gamma'_1 \leq \gamma_2$, then $d_{G_1 \times G_2}(u_1,u_2) = d_{G_1}(u_1) + d_{G_2}(u_2)$.

Proof:

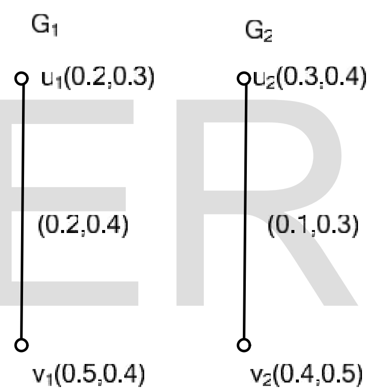
From (1),

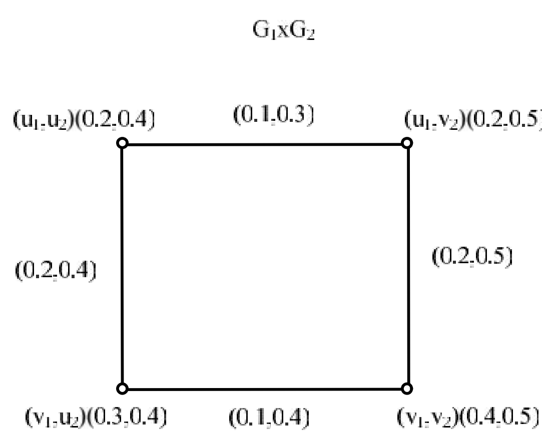
$$d_{G_1 \times G_2}(u_1,u_2) = \left\langle \sum_{u_1 = v_1,\,(u_2,v_2) \in E_2} \mu_1(u_1) \wedge \mu'_2(u_2,v_2), \sum_{u_1 = v_1,\,(u_2,v_2) \in E_2} \gamma_1(u_1) \vee \gamma'_2(u_2,v_2)\right\rangle + \left\langle \sum_{u_2 = v_2,\,(u_1,v_1) \in E_1} \mu'_1(u_2) \wedge \mu_2(u_1,v_1), \sum_{u_2 = v_2,\,(u_1,v_1) \in E_1} \gamma'_1(u_2) \wedge \gamma_2(u_1,v_1)\right\rangle.$$

$$= \left\langle \sum_{u_1 = v_1} \mu'_2(u_2,v_2), \sum_{u_1 = v_1} \gamma'_2(u_2,v_2)\right\rangle + \left\langle \sum_{u_2 = v_2} \mu_2(u_1,v_1), \sum_{u_2 = v_2} \gamma_2(u_1,v_1)\right\rangle.$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1).$$

3.1 Example





Here, $d_{G_1 \times G_2}(u_1,u_2) = (0.3, 0.7)$.

$$d_{G_1}(u_1)+d_{G_2}(u_2)=(0.2,0.4)+(0.1,0.3) = (0.3,0.7)$$

3.2 Theorem

If $G_1:\langle(v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\rangle$ and $G_2:\langle(v_i,\mu'_1,\gamma'_1),(e_{ij},\mu'_2,\gamma'_2)\rangle$ are two IFGs such that $\mu_1 \le \mu'_2$ & $\gamma_1 \ge \gamma'_2$, then $\mu'_1 \ge \mu_2$ & $\gamma'_1 \le \gamma_2$ and vice versa.

Proof:
By the definition of intuitionistic fuzzy graphs,

$$\mu_2(v_i,v_j) \le \mu_1(v_i)\wedge\mu_1(v_j) \text{ and}$$
$$\gamma_2(v_i,v_j) \le \gamma_1(v_i)\vee\gamma_1(v_j).$$

Therefore all $\mu_2 \le \max\mu_1$ and $\min\mu_2 \le \mu_1$; also all $\gamma_2 \ge \min\gamma_1$ and $\max\gamma_2 \ge \gamma_1$

Also, since $\mu_1 \le \mu'_2$, $\max\ \mu_1 \le \min\ \mu'_2$; and Since $\gamma_1 \ge \gamma'_2$, $\min\ \gamma_1 \ge \max\ \gamma'_2$.

Hence $\mu_2 \le \max\mu_1 \le \min\mu'_2 \le \mu'_1$ and $\gamma_2 \ge \min\gamma_1 \ge \max\gamma'_2 \ge \gamma'_1$.

Thus $\mu'_1 \ge \mu_2$ & $\gamma'_1 \le \gamma_2$

3.3 Theorem

Let $G_1:\langle(v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\rangle$ and $G_2:\langle(v_i,\mu'_1,\gamma'_1),(e_{ij},\mu'_2,\gamma'_2)\rangle$ be two IFGs.

(i) If $\mu_1 \le \mu'_2$ & $\gamma_1 \ge \gamma'_2$ and μ_1 & γ_1 are constant functions say C_1 and C_2 respectively. Then

$$d_{G_1\times G_2}(u_1,u_2)=d_{G_1}(u_1)+\left\langle c_1 d_{G_2^*}(u_2), c_2 d_{G_2^*}(u_2)\right\rangle$$
.

(ii) If $\mu'_1 \le \mu_2$ & $\gamma'_1 \ge \gamma_2$ and μ'_1 & γ'_1 are constant functions say C_1 and C_2 respectively. Then

$$d_{G_1\times G_2}(u_1,u_2)=d_{G_2}(u_2)+\left\langle c_1 d_{G_1^*}(u_1), c_2 d_{G_1^*}(u_1)\right\rangle$$

Proof:

(i) we have $\mu_1 \le \mu'_2$ & $\gamma_1 \ge \gamma'_2$. Hence by theorem 2, $\mu'_1 \ge \mu_2$ & $\gamma'_1 \le \gamma_2$.

From (1),

$d_{G_1\times G_2}(u_1,u_2)$ =

$$\left\langle \sum_{u_1=v_1,\,(u_2,v_2)\in E_2}\mu_1(u_1)\wedge\mu'_2(u_2,v_2),\ \sum_{u_1=v_1,\,(u_2,v_2)\in E_2}\gamma_1(u_1)\vee\gamma'_2(u_2,v_2)\right\rangle +$$
$$\left\langle \sum_{u_2=v_2,\,(u_1,v_1)\in E_1}\mu'_1(u_2)\wedge\mu_2(u_1,v_1),\ \sum_{u_2=v_2,\,(u_1,v_1)\in E_1}\gamma'_1(u_2)\wedge\gamma_2(u_1,v_1)\right\rangle.$$

$$=\left\langle \sum_{u_2v_2\in E_2}\mu_1(u_1),\ \sum_{u_2v_2\in E_2}\gamma_1(u_1)\right\rangle +$$
$$\left\langle \sum_{(u_1,v_1)\in E_1}\mu_2(u_1,v_1),\ \sum_{(u_1,v_1)\in E_1}\gamma_2(u_1,v_1)\right\rangle$$

$$=\left\langle \sum_{u_2v_2\in E_2}c_1,\ \sum_{u_2v_2\in E_2}c_2\right\rangle +$$
$$\left\langle \sum_{(u_1,v_1)\in E_1}\mu_2(u_1,v_1),\ \sum_{(u_1,v_1)\in E_1}\gamma_2(u_1,v_1)\right\rangle$$

Since μ_1 & γ_1 are constant functions

$$=\left\langle c_1 d_{G_2^*}(u_2), c_2 d_{G_2^*}(u_2)\right\rangle + d_{G_1}(u_1).$$

(ii) we have $\mu'_1 \le \mu_2$ & $\gamma'_1 \ge \gamma_2$. Hence by theorem 2, $\mu_1 \ge \mu'_2$ & $\gamma_1 \le \gamma'_2$

From (1),

$d_{G_1\times G_2}(u_1,u_2)$ =

$$\left\langle \sum_{u_1=v_1,\,(u_2,v_2)\in E_2}\mu_1(u_1)\wedge\mu'_2(u_2,v_2),\ \sum_{u_1=v_1,\,(u_2,v_2)\in E_2}\gamma_1(u_1)\vee\gamma'_2(u_2,v_2)\right\rangle +$$
$$\left\langle \sum_{u_2=v_2,\,(u_1,v_1)\in E_1}\mu'_1(u_2)\wedge\mu_2(u_1,v_1),\ \sum_{u_2=v_2,\,(u_1,v_1)\in E_1}\gamma'_1(u_2)\wedge\gamma_2(u_1,v_1)\right\rangle.$$

$$=\left\langle \sum_{(u_2,v_2)\in E_2}\mu'_2(u_2,v_2),\ \sum_{(u_2,v_2)\in E_2}\gamma'_2(u_2,v_2)\right\rangle$$
$$+\left\langle \sum_{(u_1,v_1)\in E_1}\mu'_1(u_2),\ \sum_{(u_1,v_1)\in E_1}\gamma'_1(u_2)\right\rangle.$$

$$= \left\langle \sum_{(u_2,v_2)\in E_2} \mu_2^{'}(u_2,v_2)\ ,\ \sum_{(u_2,v_2)\in E_2} \gamma_2^{'}(u_2,v_2) \right\rangle$$

$$+\left\langle c_1 d_{G_1^*}(u_1),\ c_2 d_{G_1^*}(u_1)\right\rangle$$

$$= d_{G_2}(u_2)+\left\langle c_1 d_{G_1^*}(u_1),\ c_2 d_{G_1^*}(u_1)\right\rangle.$$

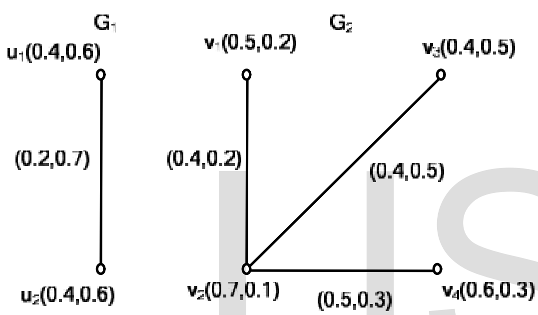
3.2 REMARK

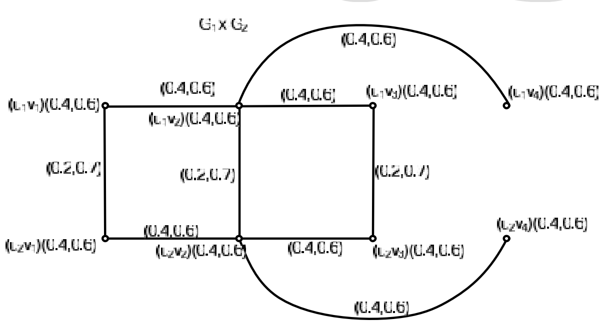
If $c_1 = c_2$, then in theorem 3.3

(i) $d_{G_1\times G_2}(u_1,u_2) = d_{G_1}(u_1) + c d_{G_2^*}(u_2)$ and

(ii) $d_{G_1\times G_2}(u_1,u_2) = d_{G_2}(u_2) + c d_{G_1^*}(u_1)$

3.1 Example





Here, $d_{G_1\times G_2}(u_2,v_2) = (1.4, 2.5)$

$$d_{G_1}(u_2) + \left\langle c_1 d_{G_2^*}(v_2), c_2 d_{G_2^*}(v_2)\right\rangle = (0.2,0.7) + \langle 0.4(3)\ +\ 0.6(3)\rangle$$
$$= (1.4,\ 2.5)$$

4 CONCLUSION

Using the concepts of Intuitionistic fuzzy graph the vertex degree cartesion product of two IFG under some condition is defined in this paper. The result has been illustrated through some examples. When the product of two graphs is larger in structure these concepts will be helpful to analyze the vertex degree without constructing the entire structure. Future work can be done to use this concept in the application of network analysis and pattern clustering.

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